Cover picture:
crack pattern of a masonry barrel vault.

TRADITIONAL TOOLS AND MODERN TECHNOLOGIES FOR THE ANALYSIS OF MASONRY STRUCTURES:

the case of the Church of Saint Andrea in Anagni

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ABSTRACT

The assessment of the structural state of historical masonry buildings is a theme of urgent necessity for preserving the Italian and European Cultural Heritage. In this sense, this study, carried out in the framework of the 3rd International Summer School on Historic Masonry Structures (Anagni, 2021), intends to provide a schemat-

ic guideline for conducting preliminary analysis to evaluate the static behaviour of historic masonry buildings. In particular, the present work focuses on the case study of the Church of Saint Andrea in the historical centre of Anagni.
The structural state of the church's principal elements is established starting from historical analysis and integrating with an accurate survey of the current crack pattern. A detailed survey carried out using the latest technology provides an accurate description of the church's geometry: of the exterior and the interior, which are used to study the state of the main arch, of the vault, and of the lateral lunettes. The analyses have been executed adopting traditional tools of the graphical statics, i.e., using Thrust Line (TL) approach and under the hypothesis of the Limit Analysis for rigid no tension materials. In this manner, no software is needed, and the structural state can be easily estimated. Without any claim of completeness for the case study, this work stands as a paradigmatic guide for people who intend to approach the stability check of historical buildings, integrating modern technologies with traditional instruments.

SOMMARIO
STRUMENTI TRADIZIONALI E TECNOLOGIE MODERNE PER L'ANALISI DI STRUTTURE IN MURATURA: il caso della chiesa di Sant'Andrea in Anagni

La valutazione dello stato di conservazione degli edifici storici in muratura riveste un ruolo centrale nell'ambito della protezione del patrimonio culturale italiano ed europeo. A tale riguardo, il presente studio, nato nell'ambito della 3a edizione dell'International Summer School on Historic Masonry Structures (Anagni, 2021), intende fornire una linea guida schematica per la conduzione di analisi preliminari per la valutazione del comportamento statico di edifici storici in muratura. In particolare, il lavoro si concentra sul caso studio della Chiesa di S. Andrea, sita nel centro storico di Anagni.
Lo stato strutturale in cui riversano i principali elementi costruttivi della chiesa viene stabilito partendo dall'analisi storica, integrata con un accurato rilievo dell'attuale quadro fessurativo. Un dettagliato rilievo dell'involucro esterno e dei locali interni del manufatto, effettuato con le più moderne tecnologie, fornisce una descrizione accurata della geometria della chiesa, e viene adottato per studiare la sicurezza statica di un arco principale, di una volta a botte e di lunette laterali. Le analisi sono state eseguite adottando gli strumenti tradizionali della statica grafica, ossia l'approccio della curva delle pressioni, sotto le ipotesi dell'Analisi Limite per materiali rigidi non resistenti a trazione. Tale metodo consente di stimare lo stato strutturale degli elementi, senza ricorrere ad alcun software di calcolo. Senza alcuna pretesa di completezza per il caso di studio, questo lavoro si pone come una guida paradigmatica per chi intende avvicinarsi alla verifica di stabilità degli edifici storici, integrando le moderne tecnologie con gli strumenti tradizionali.

KEYWORDS | PAROLE CHIAVE
International Summer School on Historic Masonry Structures, Structural assessment of existing masonry constructions, Limit analysis, Graphic statics, masonry vaults.
International Summer School on Historic Masonry Structures, verifica strutturale delle costruzioni esistenti in muratura, Analisi limite, Statica grafica, volte in muratura.

Unreinforced masonry structures constitute a significant proportion of the Italian and European architectural heritage. The understanding of the behaviour of these structures, as well as the assessment of their stability, is a multidisciplinary topic requiring diverse expertise, from more traditional disciplines such as history, geometry, archaeology, architecture and structural engineering, to the latest surveying technologies, i.e. laser scanning, photogrammetry and aerial photogrammetry.

In this work, carried out in the framework of the 3rd International Summer School on Historic Masonry Structures, held in Anagni from 30 August to 10 September 2021, all the skills mentioned above are combined to assess the stability of some relevant parts of the Church of Saint Andrea in the historical centre of Anagni.

The main scope of the paper is to describe the methodology adopted within the courses of the Summer School to perform expeditious evaluations of the structural state of historical constructions.

The study consisted of a brief historical investigation whose scope, besides giving information on the construction date, was to trace the restorations that occurred over time, which provide information about any structural changes. Then, with the help of modern technologies, an accurate survey campaign of the structure provided precise information about the geometry of different structural elements of the church, i.e., vaults, arches, lunettes, domes and pillars.

The survey campaign also detected the crack pattern, which gave information on settlements of masonry. All this information was used as a base for the structural assessment of a few church structural elements, like a portion of a barrel vault, lunettes, arches, pillar and buttress.

Traditional graphic statics (GS) techniques were adopted for the structural investigation; in particular, within the summer school, only the Thrust Line method (TL) [1] was adopted. The validity of TL and GS methods within the context of Limit Analysis has been proven by several researchers [2,3]. The TL method developed for evaluating arches [1] has also been applied by using the slicing technique to assess the structural state of barrel vaults [4], domes [5-7] and historical bridges [8]. Indeed, the slicing technique allows for assessing the equilibrium of spatial structure by dividing them into slices, analyzing each one as an arch [9-10]. In the present research, the slicing technique has been adopted to estimate the state of the barrel vault analyzed, which was imagined as a set of parallel arches.

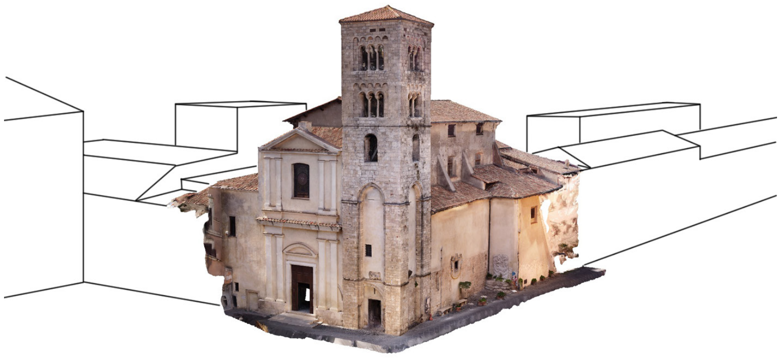
In the scientific literature, several numerical advanced tools can be applied to evaluate the equilibrium of vaults and domes [11-17]; however, their application goes beyond the scope of the present work, and it requires advanced skills and the aid of a computational tool.

Far from being a structural analysis of a whole structure, this work represents a methodological guide for studying masonry structures, which integrates the traditional methods adopted by architects and engineers along the centuries with contemporary technologies from which also structural assessment takes advantage.

The authors also remark that this study aims to show the methodology adopted in the two-week workshop, in which young researchers with different backgrounds learned the methodology and GS tools and applied them to estimate the structural state of Saint Andrea.

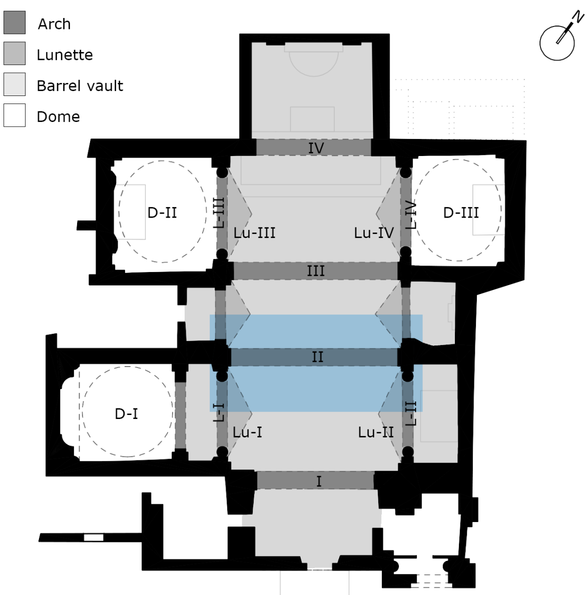
OVERVIEW OF SAINT ANDREA CHURCH IN ANAGNI

The church of Saint Andrea, built during the 11th century, is one of the historical churches of the medieval town of Anagni (Frosinone, Italy). The church is oriented in the northwest-southeast direction and presents a bell tower placed close to the southeast corner. The north and west sides of the church are attached to other buildings; thus, only the south and the east facades are entirely visible (figure 1).



1. South-est perspective view of Saint Andrea. Photogrammetry reconstruction.

Nowadays, the church presents an irregular plan with one nave, a small transept, and several chapels located at the sides of the nave. Due to many changes and additions throughout history, the geometrical makeup of the church is an overlapping of different shapes and volumes (figure 2). Upon entry, one stands in a 19 meters long nave, which is the predominant part of the church, adjacent to which are attached secondary rooms. An almost perfectly circular barrel vault of 9.8 meters in diameter covers the central nave. It is divided into several sections, of uneven length, by four transversal arches of 1 meter width, denoted from now on by I, II, III, IV. Those arches are supported by eight columns connected to perpendicular walls. Apart from acting as buttresses, the walls define six different spaces, three on each side of the nave.



2. Plan view of Saint Andrea. Highlighted in blue is the portion analyzed within the research.

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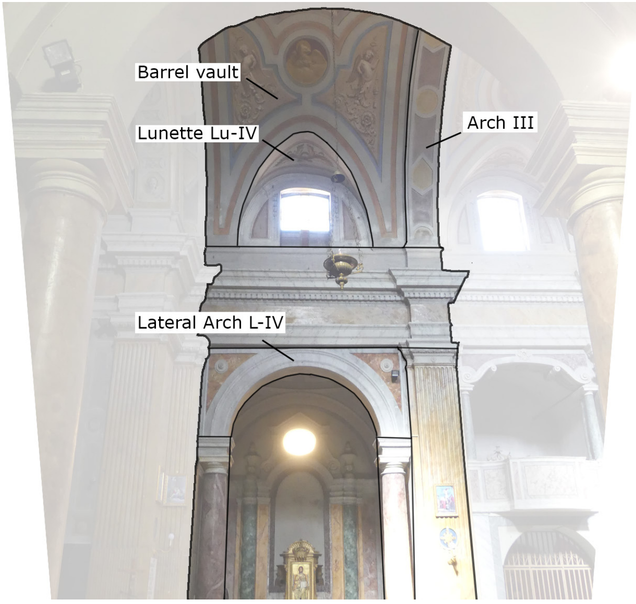
Five of them are secondary chapels, and three present oval domes (D-I, D-II and D-III). Interestingly, each chapel has a different dimension, a different vault, and a different roof structure. Lateral arches denoted with the L-I, L-II, L-III and L-IV, and other pillars mark each passage from the nave to the lateral spaces, as the chapels or secondary rooms. Further lunettes, whose dimensions are about 3.7 meters in width and 2.7 meters high, connect the nave's barrel vault with the lateral arches that mark the lateral spaces (figure 3).



3. Interior view of Saint Andrea Church. The organization of the structural elements.

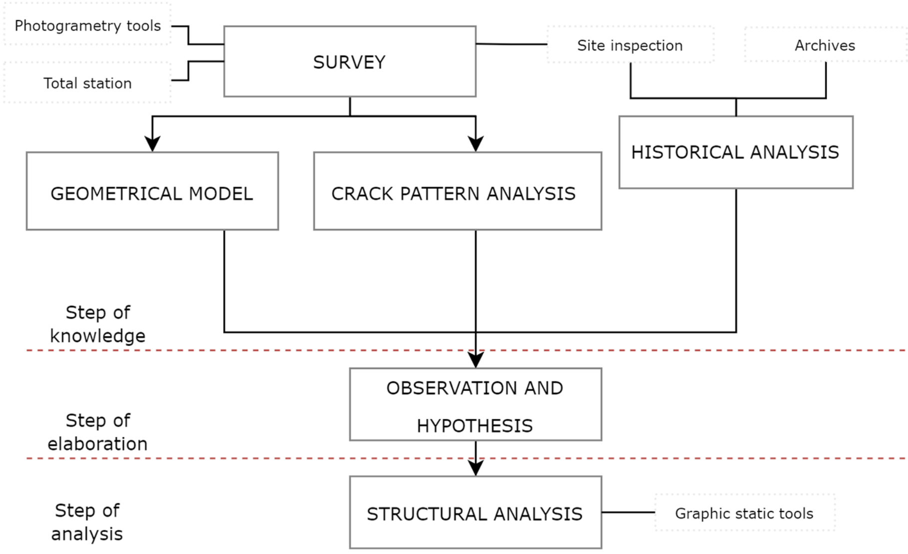
METHODOLOGY

The methodology applied to perform the structural analysis executed within the research can be summarized through a few steps (figure 4). The first step is the acquisition of the necessary information. With this aim, on-on-site inspections and a detailed survey campaign are performed. The survey was conducted with a focus on describing the building, its structural components and detecting damages and cracks. Further, the detailed survey permits obtaining a geometrical model of the church, in which settlements, cracks and other damages are represented. These are essential to explain the state of the structure, as in the adopted case study. Further, the knowledge of the building's history is necessary for performing a coherent analysis. In Saint Andrea, for example, the presence of a crypt and the knowledge of the historical structural changes led to identifying the reason for the crack pattern located in the north portion of the church. Thus, aiming at the structural assessment, the historical analysis should be related to the observations obtained in the on-site inspection and to the crack pattern analysis.

The second step concerns the elaboration of the notions acquired; here, the hypotheses on the structural assessment are formulated. This step should be conducted only when all information collected before is sufficient, exhaustive and coherent, e.g., the lack of compatibility between historical notions and crack patterns could denote that some phenomena have not been acquired, leading to wrong assumptions, or worse, neglecting dangerous situations.

The last step regards the structural analysis. The state of the structure is assessed considering the hypotheses formulated previously.

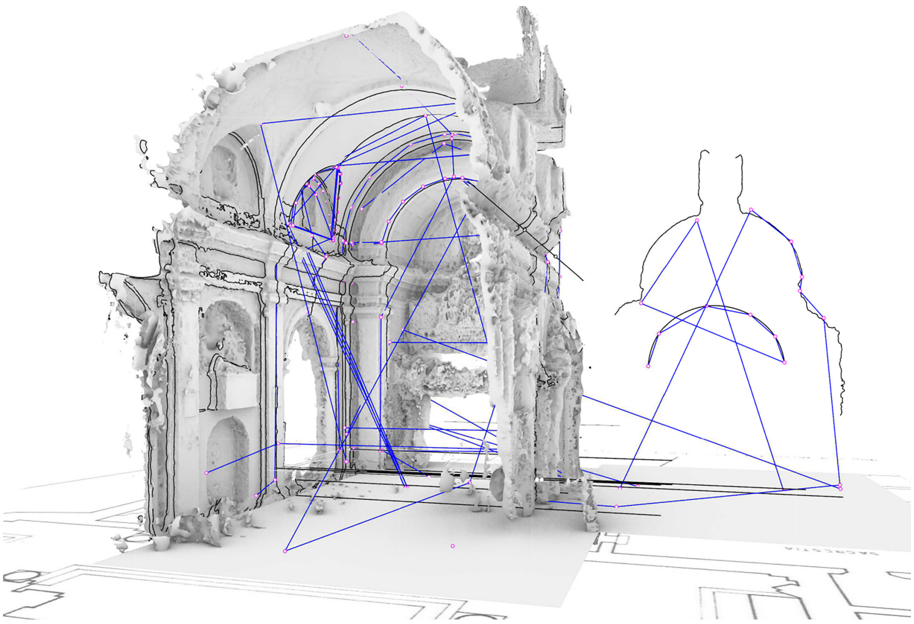
The described methodology is particularly suitable for expeditious analysis of complex structures, particularly for masonry arches, vaults and domes. Indeed, often the traditional approach requires notions impossible to know or extremely expensive in terms of time and resources.



4. Summarization of the methodology adopted for the evaluation of the structural state of Saint Andrea.

SURVEY

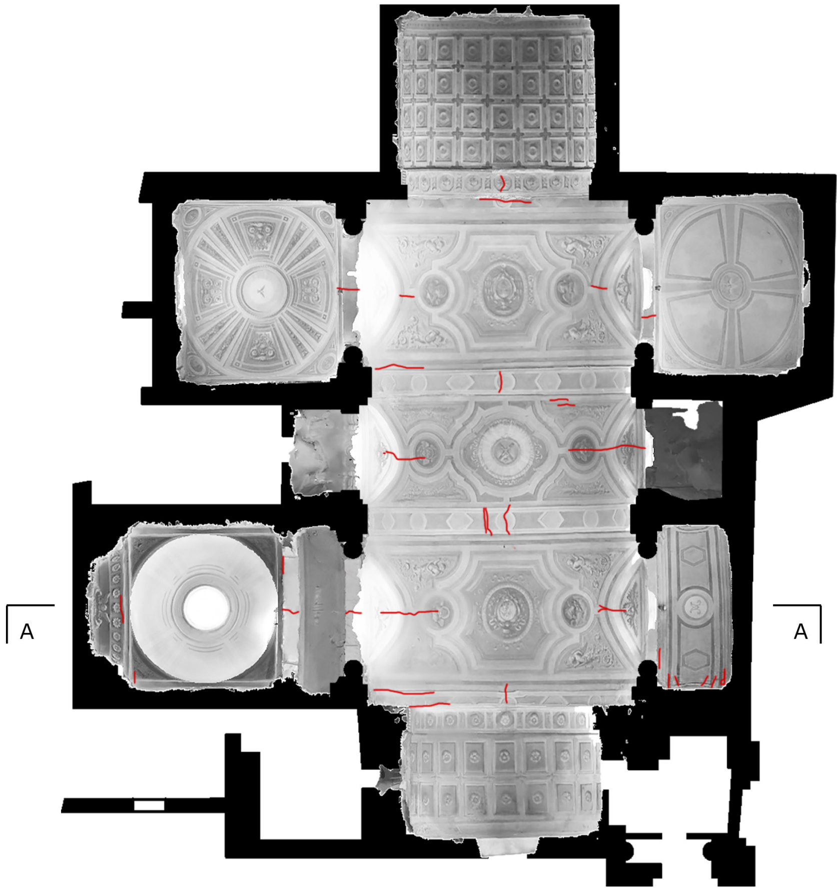
A survey campaign was carried out for acquiring the geometrical model and the crack pattern of the church, requiring both traditional and more innovative techniques. Indeed, visual inspections and field measurements were accompanied by photogrammetric and geo-referencing tools. Photogrammetry was used to acquire data at a high level of detail, whereas spatial-referencing helped to assign to the images a precise location. A 3d model, including both the exterior and the church's interior, was developed with the software Agisoft Metashape Professional (Agisoft LLC., St. Petersburg, Russia). To this scope, high-resolution photos of 4000x2250 pixels were taken. For the exterior part, a Nikon D700 camera with a 23.5mm x 15.6mm-sized CMOS sensor, 24 Megapixels, was used, and an equivalent focal length of 50mm was. The ISO sensitivity value has been set at 500, to keep digital noise to a minimum, while the diaphragm aperture was set at f/8, to have an adequate depth of field and, at the same time, to encourage the entry of sufficient light, given the poor average lighting inside the church. For the exterior part of the church, the photogrammetric survey was performed with a DJI Mavic mini drone. The drone had a 12 Megapixels camera with a CMOS sensor and a size of 6.3mm x 4.7mm. The equivalent focal length was 24mm, and the diaphragm aperture was f/2.8. In this case, photos were taken automatically, without setting any image parameters. The entire photogrammetric survey took a total of 4 hours.
The interior and the exterior models have been joined using a preliminary survey modelled through a total station (figure 5).
As described in the next section, a careful survey of the crack pattern and damages allowed to individuate those parts of the church deserving more in-depth attention and for which a specific structural analysis was needed for assessing the safety.



5. Draft of the mesh model obtained by photogrammetric survey and by total station. Highlighted in blue are triangulations executed with the total stations.

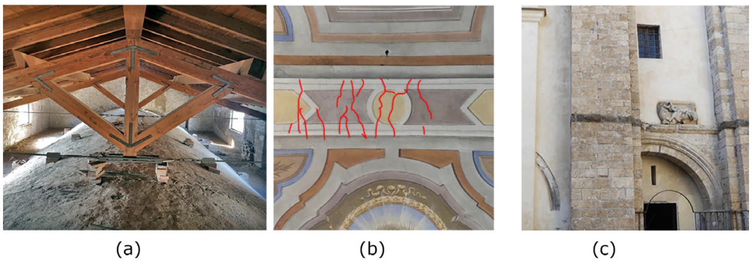
CRACK PATTERN ANALYSIS

The church of Saint Andrea has been an operational church for centuries, and like any historic structure, it has suffered some damage in the form of cracks, spalling, and consequences produced by humidity and water penetration. In particular, dampnesses are observed near the bell tower, D-I and D-II domes. A widespread crack pattern is observable in several structural elements, such as vaults, arches and walls, despite that the investigations carried out show that the church is not affected by dangerous damage.
A consistent crack pattern is generally visible along the nave, the arches and the lateral chapels (figure 6, 7). Several cracks start in the lunettes, and in some cases, they extend towards the vault of the central nave, e.g., Lu-I or Lu-II lunette. In other cases, cracks run from the arch's keystone to the base of the lunette's windows, passing through the masonry wall between them.
Thus, as in the case of the Lu-I lunette and L-I arch, a more detailed observation highlights the existence of settlements that involve several structural elements. Moreover, cracks are registered in L-I, L-II, L-III and L-IV lateral arches. The barrel vault of the nave presents a diffuse and coherent crack pattern; here, cracks are oriented along the nave's axial direction and placed in correspondence to the keystone of all arches (I, II, III and IV). Such a cracks pattern could denote an overall settlement of the structure, and the presence of hinges visible from the church's extrados could confirm this hypothesis (figure 8 (a)). Despite that, the investigations conducted from the nave's extrados have not revealed the presence of cracks or other damages. Further, no particular geometrical deformations are recorded through the survey carried out. Only in correspondence of II arch a geometric variation of the arch itself and its supporting pillar is documented. Indeed, by combining the information obtained by photogrammetric and laser scanning survey with the one executed using a total station, it has been possible to observe a geometric variation of the arch II of about 9 centimeters lower with respect to the other arches.



6. Plan view of Saint Andrea church (from bottom to top), plan view of barrel vaults, arches, domes and lunettes.

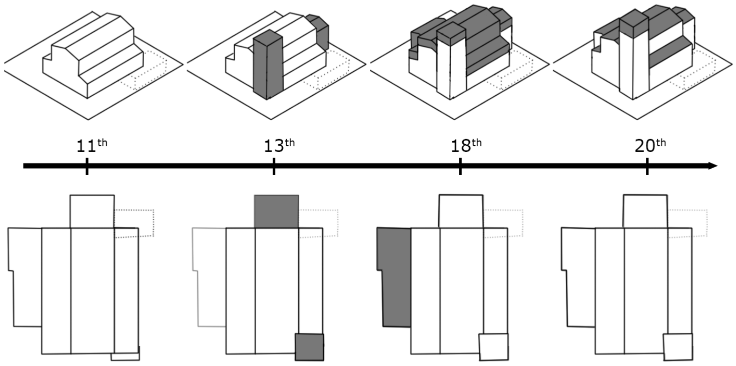
7. AA section of Saint Andrea church. Highlighted in red are cracks.



8. **(a)** extrados of barrel vault of Saint Andrea church; **(b)** intrados of the II arch; **(c)** particular of the south facade where different historical layers are still visible.

HISTORICAL ANALYSIS

As mentioned above, Saint Andrea's church presents a quite articulate geometry; the reason for its complexity lies in the countless renovations and restorations. Indeed, as often in historical churches, the Saint Andrea church has undergone at least three alterations (figure 9). The first information about Saint Andrea church dates back to the 11th century [18,19], but the presence of the pre-existing San Vito church allows us to date the church's origins even further back in time.



9. Historical evolution of Saint Andrea church. Several additions followed one another determining the current shape of the church.

In the 11th century, the church built in Romanic style had three naves covered by a classical *gable roof*. Today, different fragments of the original church are still visible in the lower part of the bell tower, particularly a portion of the vault, an arch, and a column belonging to the side nave (figure 8(c)).

The first renovation occurred during the 13th century. The church was completely renovated according to the gothic style: the bell tower was elevated, and the presbytery expanded towards the pre-existing Saint Vito church, which was incorporated into the new building as a crypt.

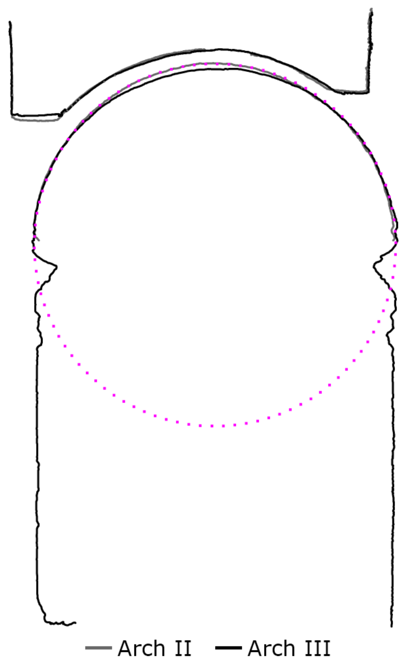
In the 18th century, the church was completely reconstructed, taking the geometry preserved until today, i.e., with a unique nave and lateral chapels, according to Baroque architecture. The church was also elevated by one-third of its previous height, laterally expanded, and the lateral Romanic vaults demolished. The south facade was elevated between 1761 and 1764, as well as the parish house was built.

In more recent times, during the 20th century, the bell tower was restored, reinforced and elevated. Indeed, diffuse cracks occurred mainly in its upper portion due to the precedent elevation. After this intervention, several chains were placed to prevent the opening of the facade and the bell tower. At last, the roof was restored and replaced with a new wooden one, with the addition of a reinforced concrete curb.

OBSERVATION AND HYPOTHESIS

The historical alteration over centuries of the church of Saint Andrea could explain the crack pattern described above (figure 6). Two historical changes are deemed to have a non-negligible structural influence: the enlargement of the presbytery with the elevation of the bell tower that occurred in the 13th century; and the renovation of the 18th century. Thus, the church was considerably enlarged and elevated in both renovations, probably leading to the formation of the crack pattern observable today. From historical investigations, the original Romanic vaults were demolished and additional buildings were inglobated to the Saint Andrea Church. In this logic, the cracks observed in the presbytery zone can be reasonably associated with the structural enlargement and the following settlements, as well as the big transversal crack located on the top of arch IV and the longitudinal crack on arch III. Indeed, potential geotechnical issues derived from elevations or geometric alterations of the structure could have induced differential settlements and thus led to diffuse cracks. Analogue reasons can justify the vertical cracks (figure 7) present in the secondary chapel connected to the bell tower and the one observable on the front-façade. These vertical cracks are typical examples of the detachment of structural elements built in different periods and not adequately connected [20]. Also, the keystone crack probably was caused by additional structures built for the bell tower's elevation.

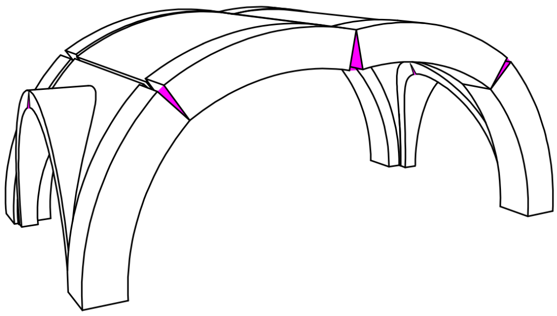
All arches I, II, III, IV have cracks in the keystone, but only arch II shows a significant geometrical alteration. Compared to a circle traced using arches I and III (figure 10), arch II presents a lowering of about 9 centimeters near the keystone and on the west side. Despite that, no cracks related to its settlement concerning other structural elements are readable. Therefore, it is reasonable to think that the settlement occurred in the early stages of its construction or, more likely, before the last renovation when all visible cracks were closed, and since that, no significant movements have occurred. For this reason, the analysis carried out has focused on establishing the state of the structures located in correspondence with arch II.



10. Comparison between arch II and arch III.

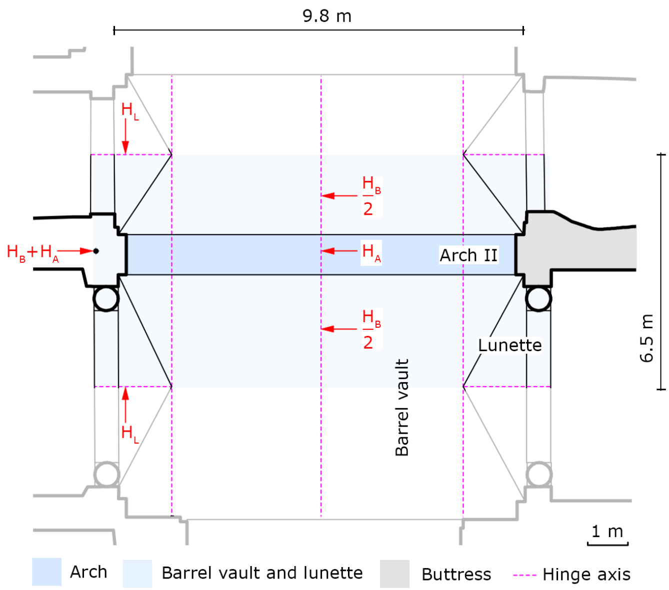
STRUCTURAL ANALYSIS

Analyses were performed assuming the existence of three hinges, whose position was compatible with the crack pattern discussed above. Thus, arch II and the portion of the barrel vault analyzed have been divided by three hinges. For both elements, the first hinge is placed at the extrados of the keystone, and the other two, symmetrically placed, are positioned at the intrados near the lunettes keystone (figure 11).



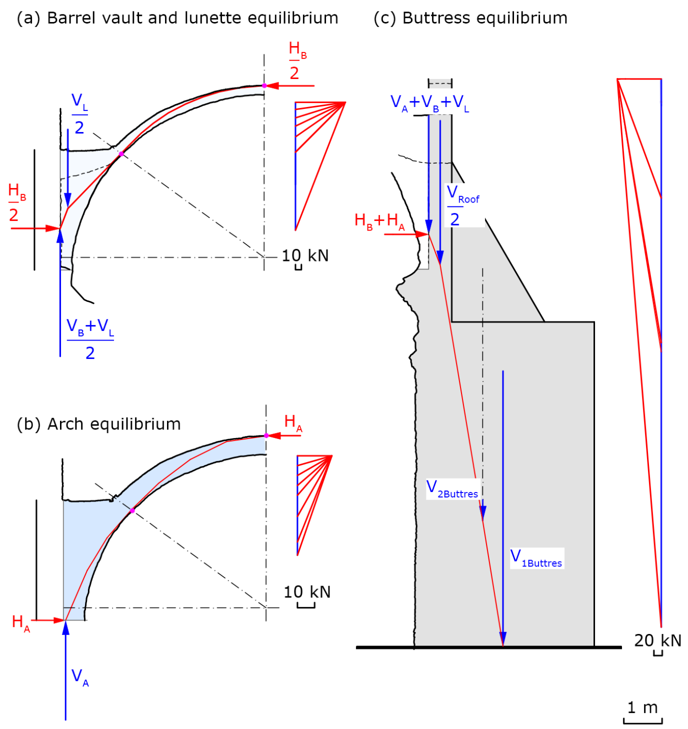
11. Model of the analyzed structure. Highlighted in purple are the hinges.

In this manner, arch II and the portion of barrel vault analyzed assume a configuration of minimum thrust, i.e. the horizontal thrust transmitted to the pillars and buttress has the minimum magnitude. As pointed out in [21], the presence of a crack does not imply instability but, more simply, a new equilibrium configuration (a three-hinge arch in this case) that allows the masonry structure to stand. Indeed, for both cracked structures, only one thrust line (TL) can pass through all three hinges, and solely if the TL entirely lies within the intrados and extrados, the equilibrium can be achieved. Thus, the portion of the Saint Andrea Church (6.5 meters long) analyzed during the investigation interests arch II, a portion of the barrel vault and lunettes. Furthermore, in order to estimate the balance of the entire critical section of the structure, the east vertical structures (pillar and buttress) on which arch II transfers loads have been also analyzed (figure 12).



12. Plan view of the structure's portion analyzed (from bottom to top).

Several horizontal thrusts act on the top of the pillar, the lunette's ones indicated with H_L, the thrust of the barrel vault indicated with H_B, and the one related to arch II labelled with H_A. Due to the geometry, the two H_L were assumed to be equal and opposite; thus, it is required to estimate H_A and H_B only.
The vault has a thickness of 25 centimeters, the arch is 38 centimeters thick, and masonry was assumed to have a specific weight of 15 kN/m³. Thus, using a graphic statics approach, the horizontal thrusts computed are H_A = 19.9 kN and H_B = 87.8 kN (figure 13). The arch's weight is 113.4 kN; thus, on each pillar, it acts V_A= 56.7kN. The vertical load related to the portion of the barrel vault and lunettes is V_B+V_L= 234.88 kN.



13. Thrust lines of barrel vault **(a)**, arch **(b)** and evaluation of the buttress equilibrium **(c)**.

The total thrust on the pillar is given by the sum of the barrel vault, lunettes and arch contributions, i.e., the horizontal component is given by H_A+H_B and the vertical one by $V_A+V_B+V_L$.
For all vertical structures (pillar and buttress), a specific weight of 18.5 kN/m³ was assumed. Therefore, the overall equilibrium has been carried out using graphic statics tools. The specific weight includes the wall's material, the roof's contribution, the fillings, and other structural components which load on the vertical elements investigated. The structural state of the structure was assessed by evaluating the Geometrical Safety Factor (GSF) (Heyman, 1966). Considering a section, the GSF is defined by the ratio between the section's half thickness and the distance from the point where the TL intersects the chosen section to the section's central point. Usually, a safe structure presents a GSF equal to 3 or higher in all sections [22]; in general, high values denote safer sections. In the case investigated at the buttress base, the GSF corresponds to 9.7. Furthermore, in all vertical structures analysed (pillar and buttress), the thrust lies within the thickness; therefore, the section is safe, and no overturning can occur.

CONCLUSION

This work described the methodology adopted by young researchers during the two-week workshops in the 3rd International Summer School on Historical Masonry Structures (Anagni, 30 August - 9 September 2021) for analyzing the structural behaviour of existing masonry constructions. The investigation focused on the case study of Saint Andrea Church in the city centre of Anagni (Frosinone, Italy). The structural behaviour has been studied using GS techniques without resorting to numerical modelling.

As the methodology above illustrated highlights, a fundamental role has been played by the survey campaign conducted with the aid of contemporary technologies, such as photogrammetry and total station. The survey, combined with the visual inspection, has furnished enough information to model a geometrical description of the church. The geometrical description, i.e., a virtual 3D model, represents the case study's actual stat, in which cracks and settlements are considered. Also, the historical analysis of the church plays a relevant role in the structural assessment since the primary goal of this investigation has been to detect structural changes due to historical renovations. All of this information has been combined to provide enough detail to formulate hypotheses on the church's behaviour. Therefore, it is possible to formulate hypotheses of the structural behaviour with contained resources through the presented methodology. This is particularly relevant for consolidation and prevention interventions to preserve cultural heritage [23].

In the context of the conducted research, the survey and the historical analysis have highlighted a peculiar configuration of arch II. Thus, the structural analysis has focused on assessing the state of the vault-arch II-pillar-buttress system from a static point of view by computing the TL and individuating the GSF. Therefore, this expeditious analysis permits to remark that the observed crack pattern on arch II has minimal relevance to the whole stability of the system.

Far from being a comprehensive guide to the structural assessment of historic buildings, this paper presents the methodology adopted in the summer school, providing an example of how an easy-to-use methodology can aid in understanding complex constructions' behaviour.

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