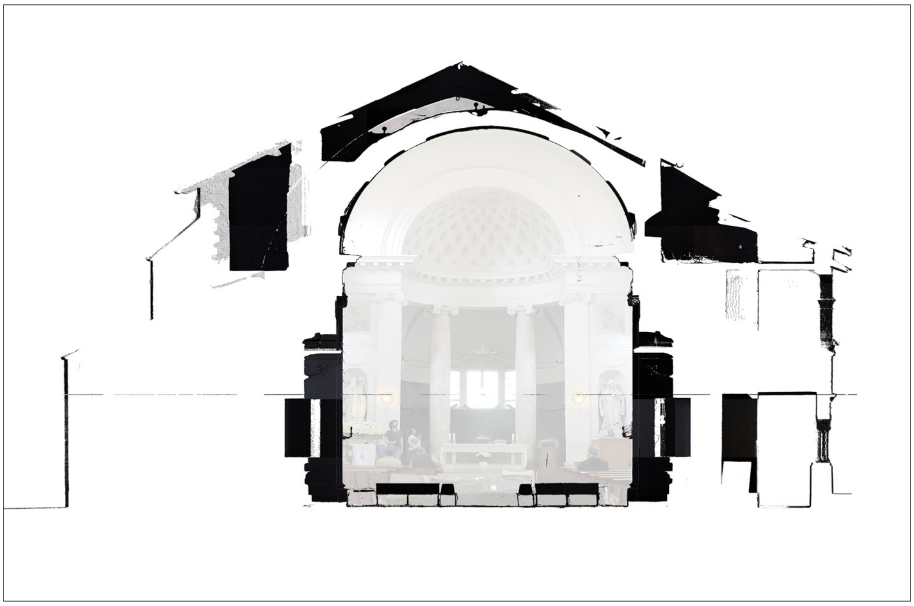


A GEOMETRIC APPROACH TO EVALUATE THE SAFETY OF MASONRY CONSTRUCTIONS:
the case of St. Scolastica Abbey in Subiaco

Nandini Priya THATIKONDA*
Università IUAV di Venezia, Italy, npthatikonda@iuav.it,
*Corresponding author

Vittorio PARIS
Università degli Studi di Bergamo, Italy, vittorio.paris@unibg.it

Concetta CUSANO
Università degli Studi della Campania "Luigi Vanvitelli", Italy,
concetta.cusano84gmail.com

Ricardo MAIA AVELINO
ETH Zürich, Switzerland, maia@arch.ethz.ch

Francesca GOBBIN
Università degli Studi di Bergamo, Italy, franci.gobbin@gmail.com

Nicola LEPORE
Politecnico di Bari, Italy, nicola.lepore@poliba.it

ABSTRACT

The paper presents the structural assessment of the neoclassical Church at the Abbey of St. Scholastica in Subiaco, located close to Rome, Italy. Here the Renaissance and gothic layers, along with the neoclassical reconstructions from 1769, are uncovered to study their role in the overall stability of the structure. With the aid of historic references, on-site visual inspection and a laser-scanning survey conducted, an understanding of the geometrical and structural configuration that defines this historic construction was established.

Cover picture: cross-sectional view of the church of St. Scholastica, highlighting the spatial asymmetry between the barrel vault and the gothic arch.

Adopting an assessment methodology based on limit analysis and graphic statics, the structural evaluation was conducted for each of the individual structural elements i.e., by isolating the arches, vaults, buttresses, and for the integrated system as a whole. Through this analysis, the Geometric Factor of Safety (GFS) related to the global equilibrium of the church's section was calculated. Without any claim of comprehensiveness in the case study, this work stands as a paradigmatic example for people intending to check the stability of historical buildings. In fact, the purpose of the research is to present, through a case study and with the aid of limit analysis approach, how the use of GFS can characterise the degree of safety in a simple and straightforward manner.

SOMMARIO
APPROCCIO GEOMETRICO PER LA VALUTAZIONE DI STRUTTURE IN MURATURA: il caso della chiesa del monastero di Santa Scolastica in Subiaco

L'articolo presenta la valutazione della sicurezza strutturale della chiesa neoclassica del monastero di Santa Scolastica di Subiaco. La compresenza di elementi gotici, rinascimentali e neoclassici è analizzata allo scopo di valutarne il ruolo nella stabilità dell'intera costruzione. A partire dall'analisi storico-critica della costruzione, da sopralluoghi in situ e accurati rilievi laser-scanner, si fornisce una più chiara comprensione della configurazione geometrica e strutturale della chiesa.
Su tale fondamento, tramite gli strumenti propri dell'Analisi Limite e della Statica Grafica, si è condotta una valutazione della sicurezza strutturale della costruzione analizzando dapprima ciascuno elemento strutturale come volte, archi e contrafforti, e infine l'intero sistema strutturale.
I risultati evidenziano come l'interazione tra le diverse stratificazioni storiche risulti di fondamentale importanza ai fini della sicurezza dell'intera costruzione. Rispetto al caso studio, il Fattore Geometrico di Sicurezza, introdotto da Heyman, è stato assunto per valutare la qualità della soluzione ammissibile trovata. Senza alcuna pretesa di completezza, la ricerca ha l'obiettivo di fornire un esempio per ricercatori e professionisti sulla possibilità di qualificare soluzioni equilibrate ottenute utilizzando gli strumenti della statica grafica.

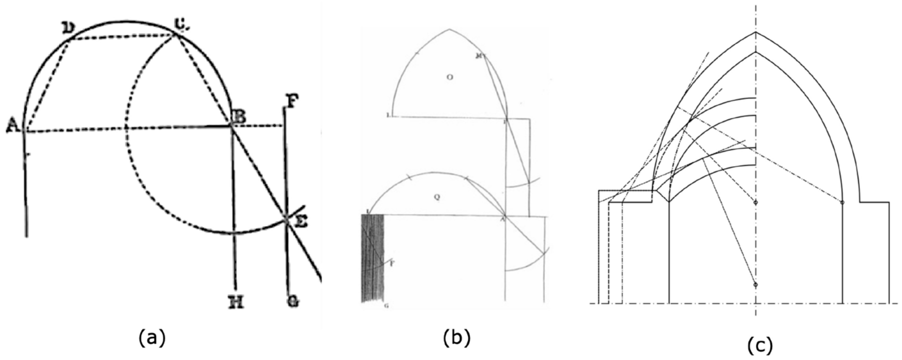
KEYWORDS | PAROLE CHIAVE
Masonry, Geometric Factor of Safety, Graphic statics, Thrust Line method, Limit Analysis
Muratura, fattore di sicurezza geometrico, statica grafica, metodo della linea di spinta, analisi limite.

The Abbey of St. Scholastica, located in the Lazio region of Italy, just outside the town of Subiaco, dates back to the 6th century. It houses buildings from different architectural styles including cosmatesque, gothic, renaissance, and neoclassical styles, and was built over ten centuries after its establishment [2].
The 1st edition of the International Summer School Historic Masonry Structures (edition of 2018) aimed at exploring approaches for the analysis of historic masonry structures and conducting a systematic study of built heritage, was strategically held at the Abbey owing to its rich, yet diverse and invaluable history. As a part of the program, four prominent structures within the complex were studied: the neoclassical church, the 14th century Gothic cloister, the refectory, and the Romanesque-style bell tower. The current study is conducted at the neoclassical church. The aim of the current research paper is to draw attention to the safety assessment of historic masonry structures, conducted using well established and simple tools based on the lower-bound theorem of limit analysis, through the use of graphic statics (GS), permitting the evaluation of Geometrical Factors of Safety (GFS) [3].
Graphic Statics allows computing the equilibrium of two-dimensional structures through a graphical representation of the forces and has been applied historically to masonry structures as in Hooke (1675) [5], Couplet (1730) [6] and Coulomb (1773) [7] [8]. Jacques Heyman presented a formulation that allows the application of limit analysis to masonry structures [1]. As such, a masonry structure can be considered safe if a compressive force path is found inside its structural geometry. The search of these internal force paths can be executed with graphic statics. The main advantage of applying graphic statics based tools to the assessment of masonry is that the analysis can be executed based purely on the geometry of the structure, in a fast and inexpensive manner [4]. The last decades have seen various advancements in this field of GS, e.g., the Trust Network Analysis (TNA) and modified TNA mainly focussing on designing and analysing three dimensional structures, such as domes or gothic vaults [9] [10] [32]. In fact, today, within the approach of limit analysis, the validity of graphic statics-based evaluation has been proven by several researchers [11] [12] [13], and often the GFS has been adopted to assess the structural safety of existing structures [31][14] [15].
Numerous other methodologies for evaluating masonry structures based on more complex approaches could be considered to estimate the safety of historical structures, like the nonlinear finite element method or the discrete element method [16]. Nevertheless, the current study resorts back to the grounded methodology of Limit Analysis conducted with the aid of graphic statics, exploiting the unilateral behaviour of masonry, and under the assumptions laid out by Heyman [1]. The research adopts a straightforward approach, to analysing historic constructions and that is highly valuable to engineers, architects and conservators working in the field of heritage preservation.
Indeed, the case study presented in this paper can reintroduce researchers and restoration experts to an important, yet simple structural assessment methodology to be applied for any built heritage under study.
The current paper is organised into four sections. The first section following the introduction provides an outline of the graphic static method and its historic evolution. Here, the concept of GFS, which is the primary coefficient for evaluation in our methodology is also introduced. The second section presents the case study, introducing the Church, its historical layering and the constructions that define it.

The third section is on the structural assessment and presents a comprehensive application of the evaluation methodology discussed in the previous sections. The research concludes with a summary of the findings and emphasises the versatile application of the adopted approach.

THE GEOMETRICAL FACTOR OF SAFETY

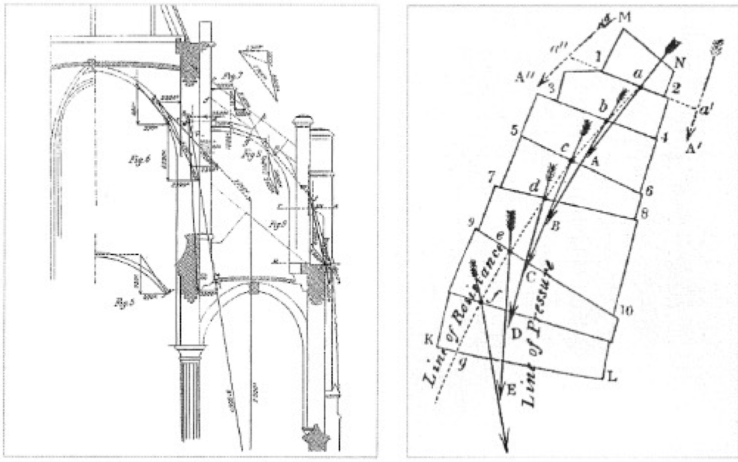
The relation between geometry and the safety of the structure has always been the primary focus of building practices; prior to the knowledge of the concept of forces and stresses, builders used geometry as a tool for building safe structures. The similarities in the proportion and relationship of different structural elements in historical buildings prove this approach's effectiveness and testify that safe and resilient buildings could be conceived even without a thorough knowledge of forces and stresses.
Between the 15th and 18th centuries, inspired by Vitruvius, numerous architectural treatises, including those of Leon Battista Alberti (1404-1472) and Andrea Palladio (1508-1580), illustrated empirical rules for the design of structures based on proportions between different structural elements [17]. Derand's rule (1643) [18] and Blondel's rule (1673) [19] are evidence of such empirical rules, where the span and rise of the arch define the thickness of the buttress. Few other gothic rules, like the one outlined by Joven Ruiz (1560) [20] even account for the arch thickness when designing a buttress. In addition, he prescribed filling of the haunches until half of the arch's rise and the minimum thickness of the arch as 1/10 of the span (figure 1.c). Though at their time, they may not have had a scientific basis for defining these rules, larger abutments were generally defined for arches that impose a significant thrust and fillings at the arches' haunches minimised this thrust imposed.



1. Geometric rules set forth for computing the thickness of the buttress. **(a)** Blondel's or Derand's rule for obtaining the buttress width, from the division of the intrados of the arch on three equal parts [19]; **(b)** application of Derand's rule to arches with different rise and span, shows the variation in the buttress thickness proposed [18]; **(c)** From the manuscript of Joven Ruiz considering the arch thickness into the geometric design of the buttress [20].

These geometric rules were specified with respect to the structural typologies, e.g., for Gothic rules they give the width of the buttresses as 1/4 of the span, or in the case of the Renaissance ones the ratio is about 1/3 - 1/2 of the span, creating much larger abutments. Similar rules are also visible for arches [3], expressing the arch's thickness as a fraction of the span, in order to be considered safe [19].
Graphic Statics (GS) offers an approach for calculating the forces in two-dimensional frames based on the relationship between its geometry and its internal forces. In GS, the internal forces are described through lines and polylines called force poly-

gons [21] (figure 2). Although the early studies related to GS date back to Hooke and Poleni's research [22], only during the 18th and 19th centuries they were widely adopted to design and assess the stability of arches, bridges and complex structures [23] [17].



(a) (b)

2. (a) Graphical analysis of equilibrium of a three-nave church (Planat, 1906): a static study of the pointed arch geometries and the supporting flying buttresses [17]; **(b)** introduction of the concept of line of thrust (Moseley, 1843) [24].

During those centuries, several researchers placed the basis for tools widely applied in the current days, such as the slicing technique formulated by Frézier (1737), today used by Huerta [15] [25]. Modern analysis tools have also been developed based on these principles such as Thrust Network Analysis [9] [31] whose roots are related to Maxwell's structural reciprocity [26], or the research conducted on the Durand-Claye method [27]. Only in 1966, Jacques Heyman provided a comprehensive methodology based on the assessment methods previously introduced but revisited it in the framework of plastic theory [1]. His formulation for estimating the state of masonry structures is based on three fundamental hypotheses:

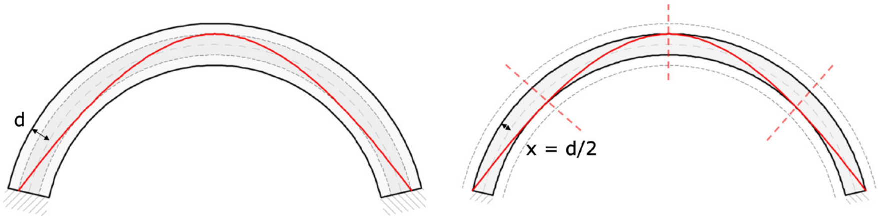
1. Sliding failure cannot occur
2. Masonry has no tensile strength
3. Masonry has an infinite compressive strength

From Heyman's hypothesis, where the masonry compressive strength is considered high enough that infinity could be assumed, it can be evident that the strength of the material has a secondary role in the safety of the masonry structure. Indeed, for ordinary masonry structures, these three hypotheses are usually respected. Thus to evaluate its behaviour, it is sufficient to focus on the relationship between the line of thrust and the structure's geometry [28].

The line of thrust is a theoretical line that can be defined as the locus of points through which the internal forces flow for a given set of loading conditions. It can be seen as the compressive path travelling within the geometry. The system's equilibrium can be visualised by drawing this line. For a given system, there could be infinite solutions to define the line of thrust, but according to the Safe Theorem, if at least one line of thrust can be found, which is in equilibrium with the external loading, and which lies entirely within the masonry envelope, then the structure can be defined as 'safe' [1].

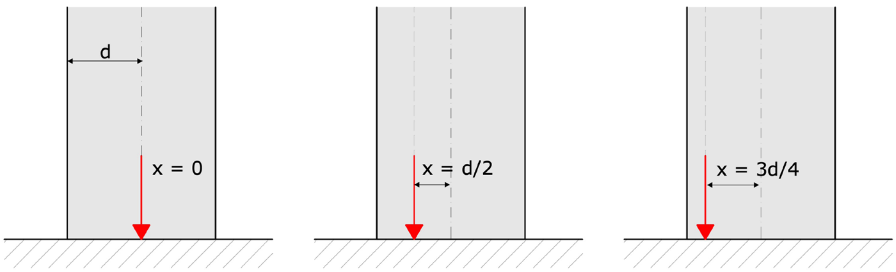
After assuming Heyman's hypothesis, the stability of masonry structures primarily depends on their geometry. Scientific backing for the empirical rules discussed and the quantification of safety was provided by Heyman (1982) [29], who introduced the concept of the geometrical factor of safety (GFS), or geometric safety factor as referred by other authors. To present the idea of GFS, a structure under a given loading scenario needs to be considered, where a line of thrust can be traced entirely within the structure. Here the GFS denotes the amount of reduction in the thickness of the structure, while still continuing to accommodate the traced line of thrust within its envelope [1]. For the case of an arch, the GFS is denoted by the ratio of the half its thickness (d) and the maximum distance between the arch's median axis and the line of thrust (x) (1), e.g., if an arch is built with a thickness of twice the minimum one, that arch would have a GFS of 2 (figure 3).

$$GFS=\frac{d}{x} \tag{1}$$



3. Geometrical factor of safety, exhibited through a segmented arch. **(a)** shows the arch, with the traced line of thrust; **(b)** shows the maximum reduction in thickness possible to continue to trace this thrust line. Here, GFS is 2, which according to Heyman [8] provides safety to avoid dangerous situations due to settlement or asymmetry.

For a footing of a buttress, the point where the line of thrust acts or where the reaction is produced defines this GFS associated with it. Similar to the computation adopted for the arch, the GFS for a buttress is the ratio of half the width of the buttress (d) to the distance of this point from the central axis (x) (figure 4).



4. Geometrical factor of safety, the case of a buttress: **(a)** the line of thrust lies precisely along the central axis of the buttress, GFS = ∞, **(b)** the line of thrust is acting at ¼ of the thickness with GFS = 2, **(c)** the line of thrust acts very close to the envelope's edge with GFS = 1.3.

As the line of thrust moves close to the envelope of the structure assessed, it can be seen that the safety associated with it reduces. On the contrary, as the eccentricity with respect to the axis of the structure minimises, the safer it is. Beyond the individual system of an arch or a buttress, the GFS can be also extended to different typologies [32].

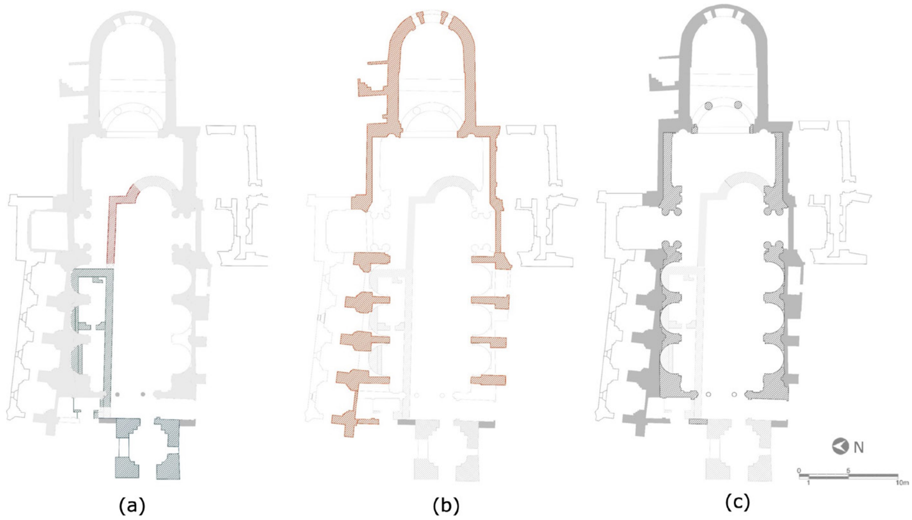
THE CASE STUDY: SAINT SCHOLASTICA CHURCH

The Abbey of Saint Scholastica was established in the 6th century AD, with its buildings arranged around three cloisters, built between the 12th and the late 16th centuries, and in three different styles, the Cosmatesque style, the Gothic style, and in the Renaissance style (figure 5). The abbey's church, the focus of the current research, is originally a Gothic structure with a Romanesque-style bell tower, entirely rebuilt in 1771–1776 in the neoclassical style [2].

(a) (b)

5. (a) The view of the Abbey of Saint Scholastica, showing the volumetric arrangement of the structures housed, **(b)** Spatial layout of the Abbey, showing the three cloisters and highlighting the church considered for evaluation.

The archival data and the remains evident at the foundation level of the Saint Scholastica Church, present the small structure which was first established here in the 6th century, and a later addition built in the 9th century and the early 11th century, with remains of a Narthex and an Apse (figure 6). Further, built in the 13th century, the superstructure is composed of five robust painted Gothic arches defining a single nave with a slightly irregular yet rectangular plan form. Each of these gothic arches spans about 12 m with a clear height of 15m. The stone masonry buttresses supporting the gothic arches are about 3.8m and 2.8m in width, on the left and right sides respectively.



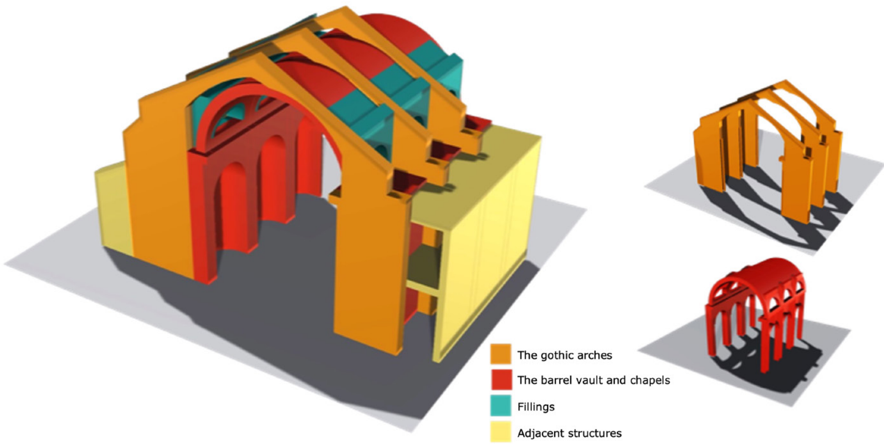
6. Historic layering of the church of St. Scholastica, **(a)** highlighting the 6th century to 11th century remains found at the foundation level, **(b)** the Gothic church built in the 13th century **(c)** the encased neoclassical church as visible in the current day.

The church as presented in the current day is the result of the modern renovation conducted in 1770 by the architect Giacomo Quarenghi in a neoclassical style [30]. The neoclassical layer is described by a barrel vault, sitting symmetrically along the axis of the apse of the existing gothic church. Spanning about 9 m in width and with 13 m of clear height under the barrel vault, this reconstruction introduced six large lunettes, three on either side of the barrel vault. Although symmetrical to the apse, this barrel vault system shows a considerable degree of asymmetry to the existing structural system: the gothic arch and buttresses, (figure 6.c), adding complexity to the understanding of how the structural system would behave.

STRUCTURAL ANALYSIS

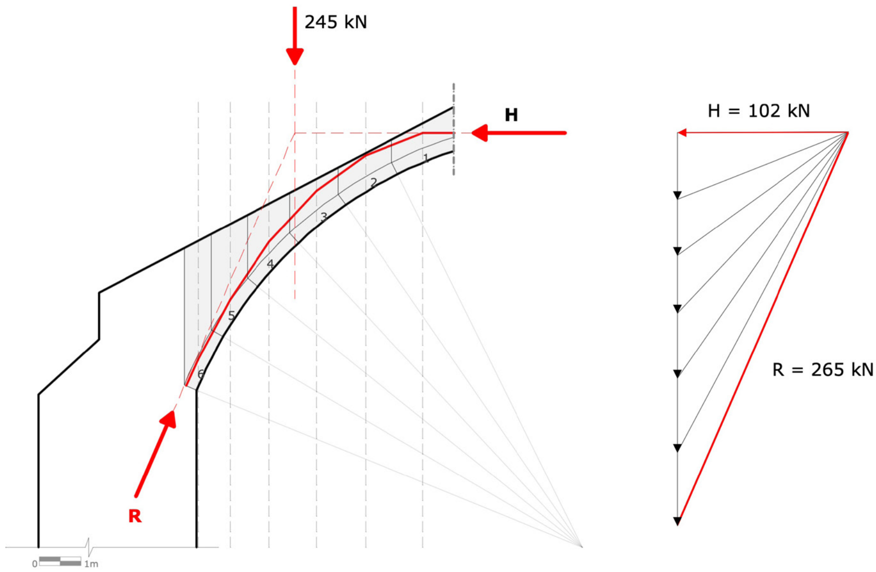
This section presents the structural analysis performed at the St. Scholastica church to evaluate its safety. It is important to remember that the church is a part of a larger aggregate; the monastery (figure 5.a), and the various interactions with its surroundings could occur, exhibiting a global structural behaviour. However, the present study disregards any interaction of the church with the surrounding constructions and analyses the central nave of the church independently. This central nave can be defined by the previously built gothic arches, the neoclassical barrel vault and filling placed on the top of the barrel vault, at its haunches. To establish a holistic geometrical understanding of these structural systems, apart from archival research, primary site inspection and laser scanning surveys were also conducted. From these inspections the three-dimensional digital model of the Church has been computed as depicted in figure 7.

In addition to defining the geometry, the surveys were also able to indicate the asymmetry present within the structure and the interaction of both the historic layers (figure 7). The inspections also capture the presence of an adjacent structure; however, these are not considered in this research due to the lack of significant structural connection with the church.



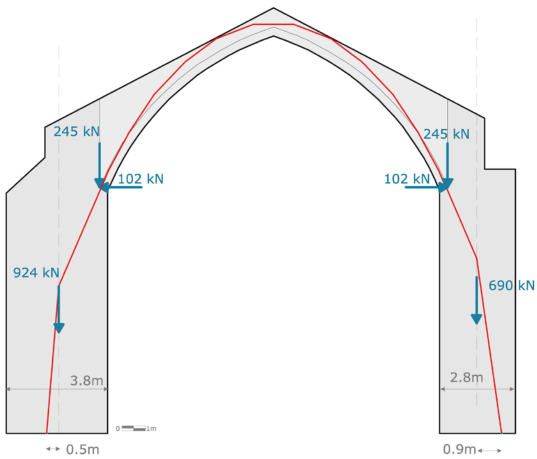
7. Digital model of the Church highlighting the main elements identified after the geometric survey system.

With a goal to individualise the contribution of the different structural systems in place, the following section presents the structural evaluation of the typical transversal section of the nave (with a width of about 1.15 m) subjected to gravity loads performed with graphic statics and under the hypothesis of Heyman's Safe Theorem. Firstly, considering the geometry of the Gothic arch alone, a line of thrust is traced. The arch has been first divided radially and then vertically to define the voussoirs (figure 8). The load contributed by the self-weight (assuming masonry density to be 20 kN/m3) of the structure and in addition, the weight of the roof was considered as the loads acting vertically over the arch, from the respective centre of mass for each voussoir. The line of thrust computed lies entirely within the arch's envelope, with the total weight transmitted down to the supporting buttress of 244 kN, thus the horizontal thrust is 102 kN (figure 8).



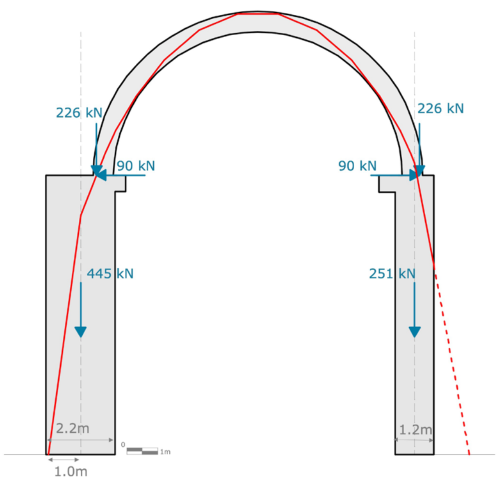
8. Computing the line of thrust for the gothic arch.

These forces transmitted by the arch are then combined with the self-weight of the buttress to assess the stability of the system. As discussed in the previous section, the GFS is computed by calculating the distance at the base of the buttress where the reaction force is produced. For the left buttresses, the reaction is at 0.45 m from the central axis, defining a GFS of 4.22 (figure 9). Similarly, for the right buttress, the reaction force is at 0.94 m from its axis, with a GFS of 1.49 associated with it. From this analysis, the existing Gothic structure is shown to be safe (with a GFS>1.0).



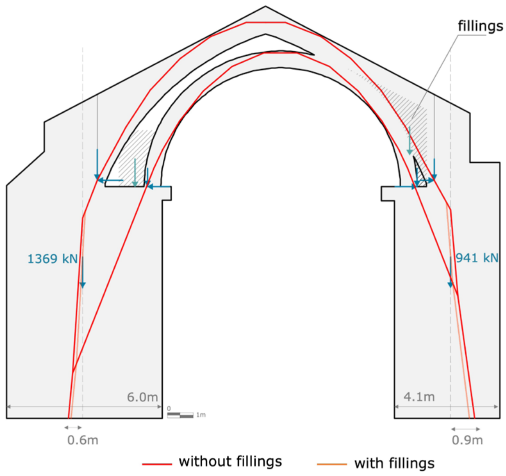
9. Computing the line of thrust for the gothic arch and buttress system.

Similar to the computation of the stability for the gothic arch and buttress system, the stability associated with the barrel vault is computed. The crosssection width and densities previously considered for the gothic arch system are adopted here as well. The horizontal thrust computed is 90 kN and the weight transmitted down to the supporting walls is 226 kN (figure 10). It should be noted that the thrust transmitted to the base of the supporting walls, including both the thrust from the arch and the self-weight of the supporting abutments and walls, shows that the line of thrust lies outside the masonry envelope (figure 10), and suggesting that the system composed of barrel vault and its supporting walls alone is not safe, with a GFS lower than 1 (right footing) and 1.07 (left footing).



10. Computing the line of thrust for the barrel vault system.

In order to evaluate the integrated behaviour of the real structural system in place, the assessment is repeated but now considering the gothic arches and the barrel vault together. The line of thrust, now, can be traced entirely within the integrated system's masonry envelope (figure 11). The reaction at the left base acts at 0.56 m from its axis, and constitutes a GFS of 5.36, whereas, on the right, the reaction is at 0.93 m from the axis with a GFS of 2.19. This analysis shows that the Renaissance intervention that occurred in the 17th century, with the addition of the barrel vault, is safe (GFS=2.19) considering the interaction with the previously built lateral system.



11. Line of thrust obtained for the integrated system, with and without the consideration of the fillings.

Further, identifying the contribution of the fillings present at the haunches of the barrel vaults is taken into consideration and this assessment is repeated. The additional load is approximately 99 kN and 116 kN on the left and right sides of the structure, respectively. This load shifts the path followed by the line of thrust, reducing its eccentricity, and improving the GFS (table 1).

	Geometric factor of safety (GFS)	
	At the footing of the left side	At the footing of the right side
Gothic Arch and buttress system	4.22	1.49
Barrel vaulting	1.07	<1.0
Gothic Arch and Barrel vault together	5.36	2.19
Gothic Arch and Barrel vault, considering the filling	6.58	2.74

Table 1. Geometric factor of safety computed for different combinations of the considered structures.

The impact of the load applied by the fillings (around 10% of the total load) is evident on both sides of the structure, with their GFS increasing by more than 15%. Furthermore, at the footing of the right side, the value is about 20%, highlighting a very important relationship between the geometry, load distribution and the GFS. As a result, the GFS of the main transversal section of the church can be taken as the lowest value considering the interaction and the presence of filling, resulting in a GFS of 2.74.

CONCLUSION

The present paper focused on a preliminary structural assessment of the Church at the Abbey of St. Scholastica in Subiaco. First, the different construction phases undergone at the Church were identified and a laser scan surveying expedition was conducted, obtaining the geometry of the Church's main structural elements. Second, the structural analysis was performed under the hypothesis of the Safe Theorem of Limit Analysis focusing on the main transversal section of the structure and considering gravity loads. The line of thrust was computed using graphic statics and the measure of the safety in the church was obtained through computing the Geometric Factor of Safety (GFS). The structural elements were analysed separately as well as an integrated system, considering different combinations. The results show that the current structure is safe and a GFS of 2.74 is obtained at the base of the smallest buttress.

Through this case study it is evident that, with minimum computational cost, the use of GFS provides a characterisation of the degree of safety for an admissible solution. Further, there is also a potential to consider the global interaction of the Church with the surrounding elements, as well as considering lateral loads, such as earthquake and wind loads, that are beyond the scope of the present paper. Therefore, without any claim of comprehensiveness, the present study stands as a paradigmatic example for engineers, architects and conservators intending to check the stability of historical buildings.

REFERENCES

[1] J. Heyman, 'The stone skeleton', Int. J. Solids Struct., vol. 2, no. 2, pp. 249–279, Apr. 1966, doi: 10.1016/0020-7683(66)90018-7.

[2] A. Ricci, Lo spazio del silenzio: storia e restauri dei monasteri benedettini di Subiaco. Tip. Ed. Santa Scolastica, 2004.

[3] S. Huerta Fernández, 'Mechanics of masonry vaults: The equilibrium approach', in Mechanics of masonry vaults: The equilibrium approach | En: Historical Constructions. Possibilities of numerical and experimental techniques | pp. 47-69 | Universidade do Minho | 2001, P. Lourenço and P. Roca, Eds. Guimaraes, Portugal: E.T.S. Arquitectura (UPM), 2001, pp. 47–69. Accessed: 07 April, 2022. [Online]. Available: https://oa.upm.es/569

[4] S. Huerta Fernández, 'Structural Analysis of Thin Tile Vaults and Domes: The Inner Oval Dome of the Basilica de los Desamparados in Valencia', in Structural Analysis of Thin Tile Vaults and Domes: The Inner Oval Dome of the Basilica de los Desamparados in Valencia | En: Nuts and Bolts of Construction History. Culture, Technology and Society | pp. 375-383 | Picard | 2012, R. Carvais, A. Guillerme, V. Nègre, and J. Sakarovitch, Eds. E.T.S. Arquitectura (UPM), 2012, pp. 375–383. Accessed: 17 March, 2022. [Online]. Available: https://oa.upm.es/11495

[5] H. Robert, A description of helioscopes, and some other instruments. London: T.R. for John Martyn, 1676. Accessed: 07 April, 2022. [Online]. Available: http://www.e-rara.ch/zut/731052

[6] P. Couplet, 'De la poussée des voûtes', Hist. L'Académie R. Sci., vol. 79, pp. 117–141, 1729.

[7] C. A. Coulomb, 'Essai sur une application des regles de maximis et minimis a quelques problemes de statique relatifs a l'architecture (essay on maximums and minimums of rules to some static problems relating to architecture)', 1973.

[8] C. A. de Coulomb and J. Heyman, Coulomb's Memoir on Statics: An Essay in the History of Civil Engineering. Cambridge University Press, 1972.

[9] P. Block and J. Ochsendorf, 'Thrust Network Analysis: A New Methodology for Three-Dimensional Equilibrium', J. Int. Assoc. Shell Spat. Struct., vol. 48, no. 3, pp. 167–173, Dec. 2007.

[10] V. Bhooshan, D. Reeves, S. Bhooshan, and P. Block, 'MayaVault—a Mesh Modelling Environment for Discrete Funicular Structures', Nexus Netw. J., vol. 20, no. 3, pp. 567–582, Dec. 2018, doi: 10.1007/s00004-018-0402-z.

[11] C. Cusano, A. Montanino, C. Olivieri, V. Paris, and C. Cennamo, 'Graphical and Analytical Quantitative Comparison in the Domes Assessment: The Case of San Francesco di Paola', Appl. Sci., vol. 11, no. 8, Art. no. 8, Jan. 2021, doi: 10.3390/app11083622.

[12] V. Paris, G. Ruscica, and G. Mirabella Roberti, 'Graphical Modelling of Hoop force Distribution for Equilibrium Analysis of Masonry Domes', Nexus Netw. J., pp. 1–20, Jun. 2021, doi: 10.1007/s00004-021-00556-x.

[13] P. Block, M. Rippmann, T. Van Mele, and D. Escobedo, 'The Armadillo Vault: Balancing computation and traditional craft', Fabricate, pp. 286–293, 2017.

[14] J. Heyman, 'On shell solutions for masonry domes', Int. J. Solids Struct., vol. 3, no. 2, pp. 227–241, Mar. 1967, doi: 10.1016/0020-7683(67)90072-8.

[15] C. Cennamo, C. Cusano, and M. Angelillo, 'A limit analysis approach for masonry domes: the basilica of San Francesco di Paola in Naples', Int. J. Mason. Res. Innov., vol. 4, no. 3, pp. 227–242, Jan. 2019, doi: 10.1504/IJMRI.2019.100568.

[16] P. Roca, M. Cervera, G. Gariup, and L. Pela', 'Structural Analysis of Masonry Historical Constructions. Classical and Advanced Approaches', Arch. Comput. Methods Eng., vol. 17, no. 3, pp. 299–325, Sep. 2010, doi: 10.1007/s11831-010-9046-1.

[17] S. Huerta, 'The Analysis of Masonry Architecture: A Historical Approach', Archit. Sci. Rev., vol. 51, no. 4, pp. 297–328, Dec. 2008, doi: 10.3763/asre.2008.5136.

[18] F. Derand, 'L'architecture des Voutes', Sébastien Cramoisy París Fr., vol. 1643.

[19] A. Gerbino, 'François Blondel and the Résolution des quatre principaux problèmes d'architecture (1673)', Soc. Archit. Hist. J., vol. 64, no. 4, pp. 498–521, 2005, doi: 10.2307/25068202.

[20] F. García Jara, La representación gráfica de bóvedas y cúpulas en el "Libro de Arquitectura" (1560) de Hernán Ruiz el Joven. Marfil, 2010. Accessed: 07 April 2022. [Online]. Available: http://rua.ua.es/dspace/handle/10045/16548

[21] P. Block, M. DeJong, and J. Ochsendorf, 'As Hangs the Flexible Line: Equilibrium of Masonry Arches', Nexus Netw. J., vol. 8, no. 2, pp. 13–24, Oct. 2006, doi: 10.1007/s00004-006-0015-9.

[22] J. Heyman and G. Poleni, 'Poleni's problem.', Proc. Inst. Civ. Eng., vol. 84, no. 4, pp. 737–759, Aug. 1988, doi: 10.1680/iicep.1988.139.

[23] W. Wolfe, Graphical analysis: a textbook on graphic statics. Sidney: McGraw-Hill book Company Inc., 1921.

[24] S. Huerta, 'The safety of masonry buttresses', Proc. Inst. Civ. Eng. - Eng. Hist. Herit., vol. 163, no. 1, pp. 3–24, Feb. 2010, doi: 10.1680/ehah.2010.163.1.3.

[25] S. Huerta, 'Oval Domes: History, Geometry and Mechanics', Nexus Network Journal, vol. 9, no. 2, pp. 211–248, Oct. 2007, doi: 10.1007/s00004-007-0040-3.

[26] W. F. Baker, L. L. Beghini, A. Mazurek, J. Carrion, and A. Beghini, 'Maxwell's reciprocal diagrams and discrete Michell frames', Struct. Multidiscip. Optim., vol. 48, no. 2, pp. 267–277, Aug. 2013, doi: 10.1007/s00158-013-0910-0.

[27] D. Aita, R. Barsottu, and S. Bennati, 'Modern reinterpretation of Durand- Claye's method for the study of equilibrium conditions of masonry domes.', in AIMETA 2017 - Proceedings of the XXIII Conference The Italian Association of Theoretical and Applied Mechanics, Mediglia (Milano), 2017, vol. 3, pp. 1459–1471.

[28] J. Heyman, 'The safety of masonry arches', Int. J. Mech. Sci., vol. 11, no. 4, pp. 363–385, Apr. 1969, doi: 10.1016/0020-7403(69)90070-8.

[29] J. Heyman, The Masonry Arch. Chichester: Ellis Horwood Ltd, 1982.

[30] P. Egidi, G. Giovannoni, and F. Herman, I Monasteri di Subiaco. Roma: Tip. dell'Unione Coop. Editrice, 1904.

[31] R. Maia Avelino, A. Iannuzzo, T. Van Mele, and P. Block, "Assessing the safety of vaulted masonry structures using thrust network analysis," Comput. Struct., vol. 257, p. 106647, Dec. 2021

[32] R. Maia Avelino, A. Iannuzzo, T. Van Mele, and P. Block, "Parametric Stability Analysis of Groin Vaults," Appl. Sci., vol. 11, no. 8, p. 3560, Apr. 2021